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DOMINGOS XAVIER VIEGAS
LUÍS MÁRIO RIBEIRO

From the probability density function of the rate of spread to that of the corresponding burned area

Alvaro Crespo-Santiago[1,2]; Gianni Pagnini*[1,3]

[1] BCAM. Bilbao, Basque Country, Spain, {gpagnini@bcamath.org , acrespo@bcamath.org}
[2] Stockholm University. Stockholm, Sweden
[3]Ikerbasque. Bilbao, Basque Country, Spain
**Corresponding author*

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Abstract

We show that the probability density function (PDF) of a burned area enclosed by a random fire-perimeter is driven by the PDF of the bounding-box sides. In particular, the random value of the area emerges to be proportional to the random position of the bounding-box sides times an averaged coefficient dependent on the geometry of the burned area. Therefore, the two PDFs are functionally equal. This means that the PDF of the burned area is driven and functionally equal to the PDF of the position of the head of the fire. The displacement of the head of the fire is given by the rate of spread (ROS), thus the PDF of the burned area results to be driven and equal to the PDF of the ROS. This result holds in general whenever the fire exhibits an advancement along a main direction. The main theoretical result has been tested by different families of stochastic processes. This study can be understood as a start for the development of a theory of stochastic dynamics of wildfire propagation with the aim, for example, to provide physically-grounded initial perturbations of wildfire perimeters for ensemble forecasting.

1. Introduction and Motivation

The determination of the spreading of wildfires is affected by large errors because of such predicting and modelling difficulties. As a consequence of this complexity, the evaluation of fire spreading predictions relies on scoring methods (Filippi 2013, Filippi 2014). Because of such unpredictability, in analogy with the weather forecast, ensemble forecasting has been formulated for wild fires, see, e.g., (Finney 2011, Allaire 2021), together with the application of data assimilation procedure, see, e.g., (Mandel 2008, Rochoux 2014, Rochoux 2015, Ferragut 2015). The idea of ensemble forecasting is based on the concept of stochastic dynamics that describes the evolution of the probability of certain observables. This limit is often associated with the rapid growth of uncertainty originating in the initial conditions; these arise both from imperfect and incomplete observations, and from inaccuracies in the formulation of the model both in terms of its physical processes and its computational representation. Ensemble prediction is a practical method to find a single, deterministic forecast with an estimate of the probability of nearby forecast states. Uncertainties in prediction by fire simulators are statistically related to the simulators' dependence on the required parameters, such that sensitivity analysis is considered for improving the reliability of predictions (Trucchia 2019, Prieto 2015, Asensio 2020) and also on the regional-scale weather prediction. Thus, such uncertainties are covered by probabilistic predictions in terms of ensemble forecasts through a proper distribution of input parameters (Allaire 2018, Allaire 2020, Allaire 2021).

The present research goes in the direction to establish the stochastic dynamics of wild fire spreading. In particular we show that the probability density function (PDF) of a burned area enclosed by a random fire-perimeter is driven by the PDF of the bounding-box sides.

2. Methods

We study the Riemann integral

$$A(t) = \int_0^{a(t)} y(x,t)\, dx,\ \ y(x,t) \geq 0\ \forall x,\ \forall t \geq 0$$

when $y(x,t)$ is replaced by a random function and $a(t)$ is replaced by a nondecreasing stochastic process. We show that, under quite general conditions in the random setting, the randomness of the outcome of the integral depends only on the randomness of the upper-bound of the integration interval such that the PDF of the random counter-part of $A(t)$ is ruled by the PDF of the random counter-part of $a(t)$. The same holds for the governing equations of the corresponding PDFs. Hence, we finally provide the PDF of the burned area in terms of the PDF of the ROS with the aim to start the development of a theory for stochastic dynamic predictions of wildfire propagation.

Observable $A(t)$ can be understood as an area that evolves in time and then its interpretation as the burned area of a wildfire is straightforward. In particular, if x-axis is aligned with the wind, the point $a(t)$ describes the motion of the head of the fire and then it is driven by the rate of spread. In the context of wildfires, the equation states that the evolution of the statistics of the burned area is determined by the evolution of the head of the fire times the mean value of the function enclosing the area. We report that that the exact shape of the function is not important and we don't account for it in this work. The perimeter of the fire perimeter is assumed to be a noisy ellipse and we use six models of random perturbations in order to capture the stochastic nature of it on each realization. Examples of the random functions enclosing the area for the six models of random perturbations are shown in Figure 1, focusing only on the first quadrant.

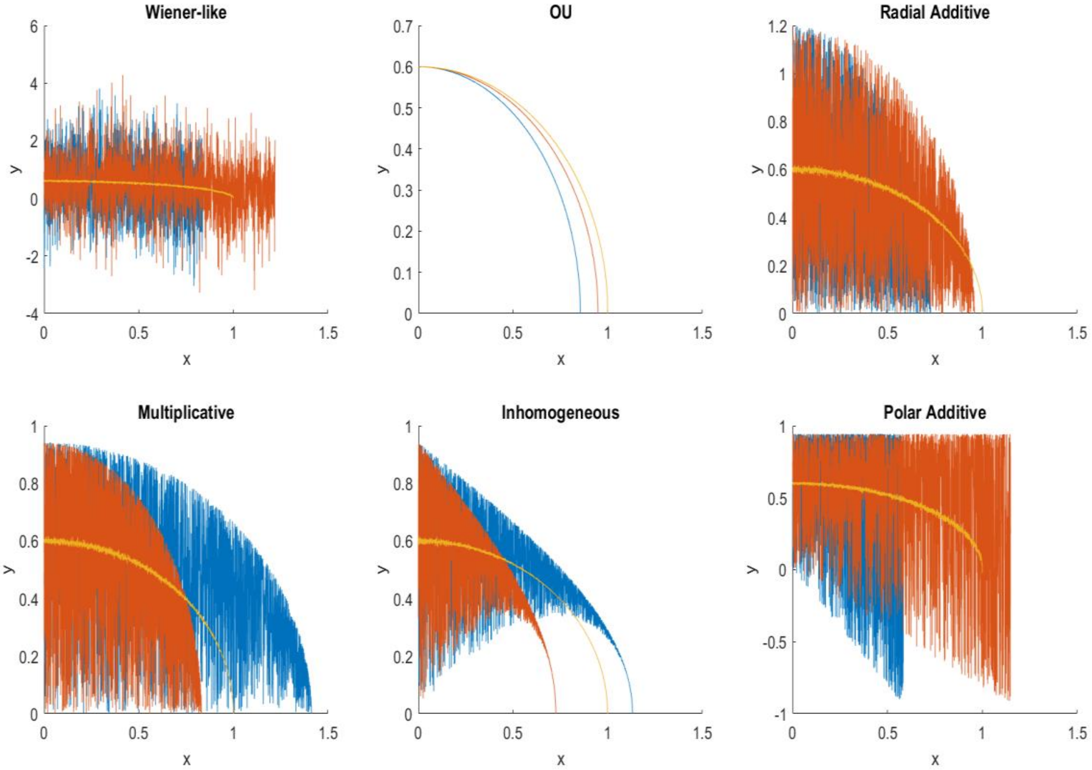

Figure 1 - Two realizations (blue and orange) of the noisy ellipse for the six random perturbations. The mean fire perimeter is also plotted (yellow).

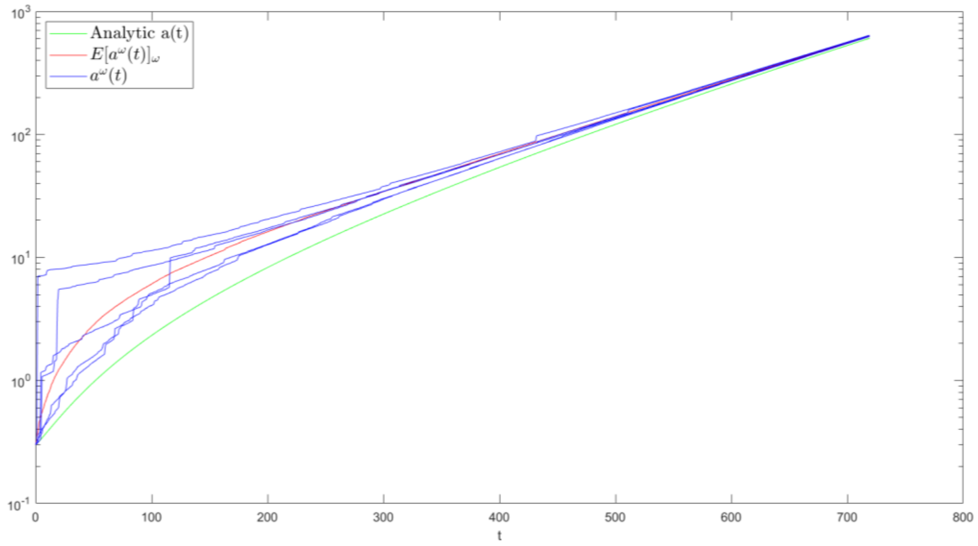

Figure 2 - Five trajectories (blue) of the head of the fire following Viegas' evolution model and lognormal perturbations and the expected value over all the realizations (red). The lowermost line (green) corresponds to the analytic value without noise. Logarithmic y-axis.

The time evolution of the head of the fire is modelized using Viegas' ROS model for eruptive fires (Viegas 2005, Viegas 2006). We use the differential equation and parameters as presented for herbaceous fuel (Viegas 2006), being R the rate of spread. The evolution of the perpendicular direction (the vertical semiaxis in the case of an elliptic-like curve) is taken as propagating with constant ROS. We take a convenient numerical value based on (Rothermel 1972) and (Catchpole 1998). For each realization a perturbation is applied to the analytical value of the head of the fire by using a lognormal PDF. All the realizations are initialized at the same point and we let them evolve as it follows. Some trajectories of the head of the fire are shown in Figure 2 together with the noise-free value and the expected value among all the realizations.

3. Results and Conclusions

We now present and analyse the qualitative results obtained by running simulations as described above and we show that the developed equality holds numerically under different conditions of the random motion of the fire front and for different PDFs of the position of the head of the fire.

First, for the six noisy-model of the front perimeters, we consider and ensemble of realizations of the head of the fire that are distributed according to a Beta distribution $B(2,2)$. In Figure 3 we see that the distribution of the area divided by the mean value of the fire front function clearly follows the distribution of the head of the fire. It can be shown that for every well-behaved PDF mapping the position of the head of the fire, the equality is fulfilled.

In particular, looking at Figure 2, we roughly distinguish three regimes on the evolution curve of the mean head of the fire: An early regime before than $t \approx 15$, a final regime after $t \approx 300$, and the transition between them. Hence, in Figures 4, 5 and 6 we show PDFs of a and $A/\overline{Y}$ for times $t = 10$, $t = 70$ and $t = 700$. We see that both PDFs are equivalent at any time, and for wildfires this shows that the statistical properties of the burned area are driven by the statistics of the position of head of the fire.

The application of this approach to real systems is limited to the existence of a main direction of advancement of the fire that is characterized by large fluctuations among independent realizations, here it is identified by a which corresponds to the head of the fire ruled by the rate of spread, and of a perpendicular direction that is characterized indeed by negligible fluctuations among independent realizations. Actually, this approach is limited, for example, to wind driven fires or fires with similar dynamics.

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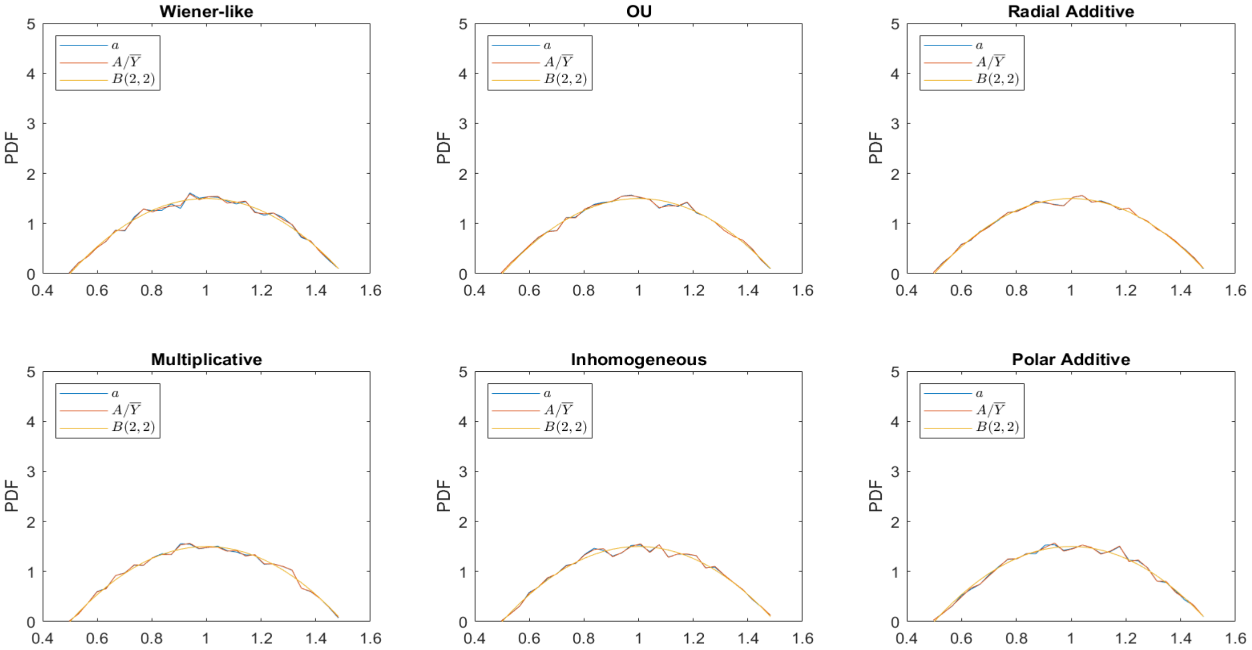

Figure 3 – Probability density functions of a and $A/\overline{Y}$, where $\overline{Y}$ is the mean value of the fire-perimeter y(x), for all the random models of fire front motion and the Beta distribution B(2,2).

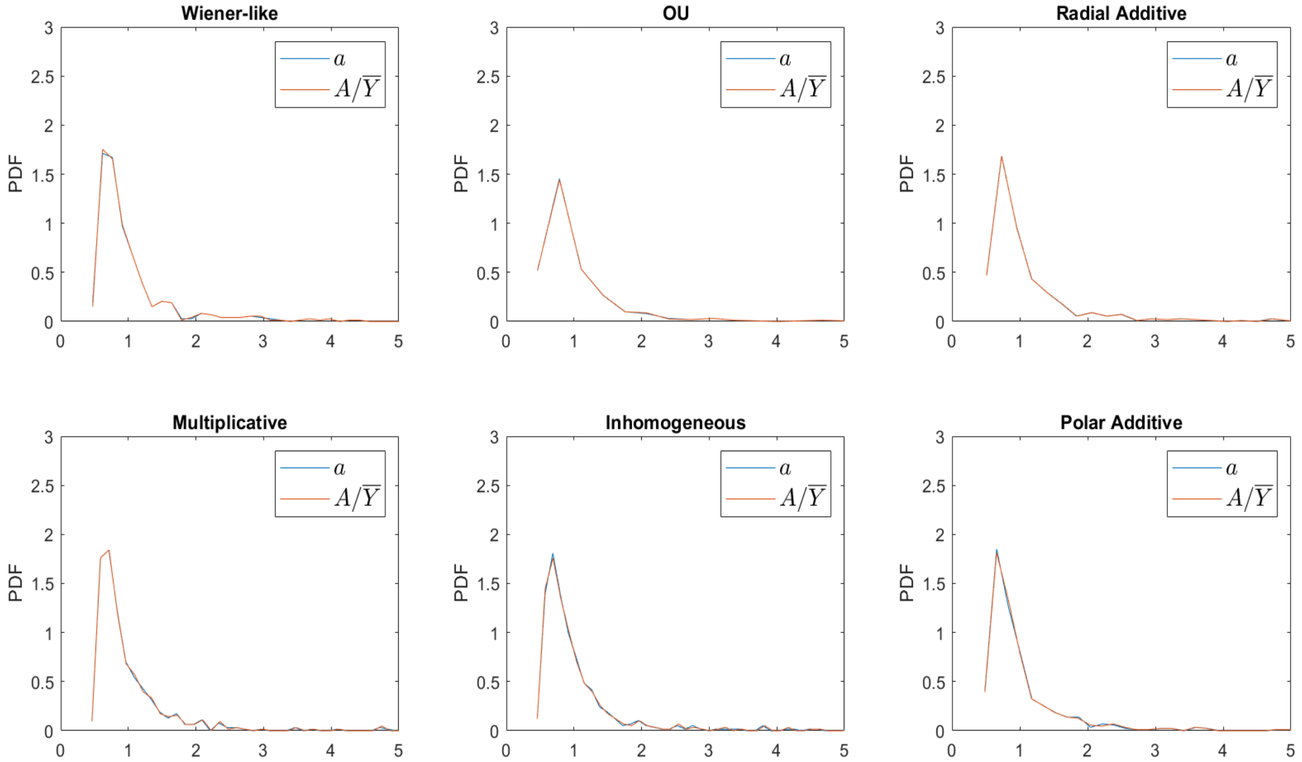

Figure 4 – Probability density functions of the head of the fire a and of the scaled burned are $A/\overline{Y}$ for all the six random models of fire front motion at time t $\approx$ 10.

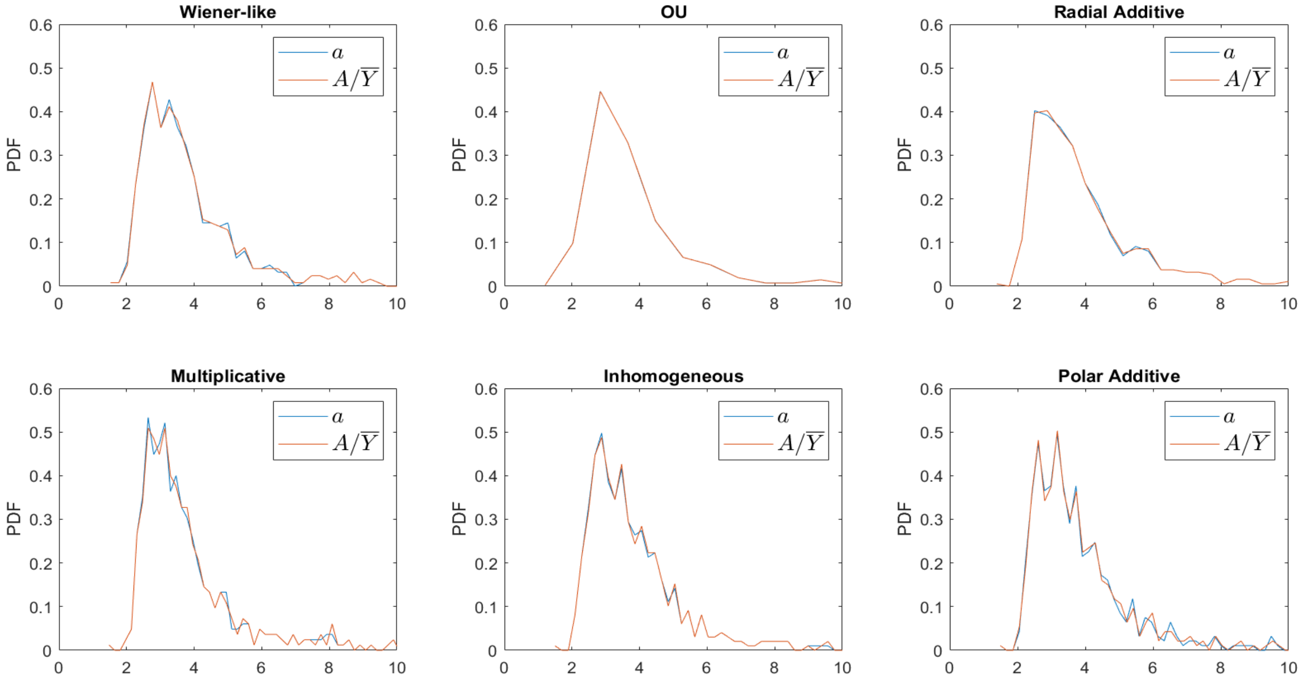

Figure 5 – Probability density functions of the head of the fire a and of the scaled burned are $A/\overline{Y}$ for all the six random fire front models at time t $\approx$ 70.

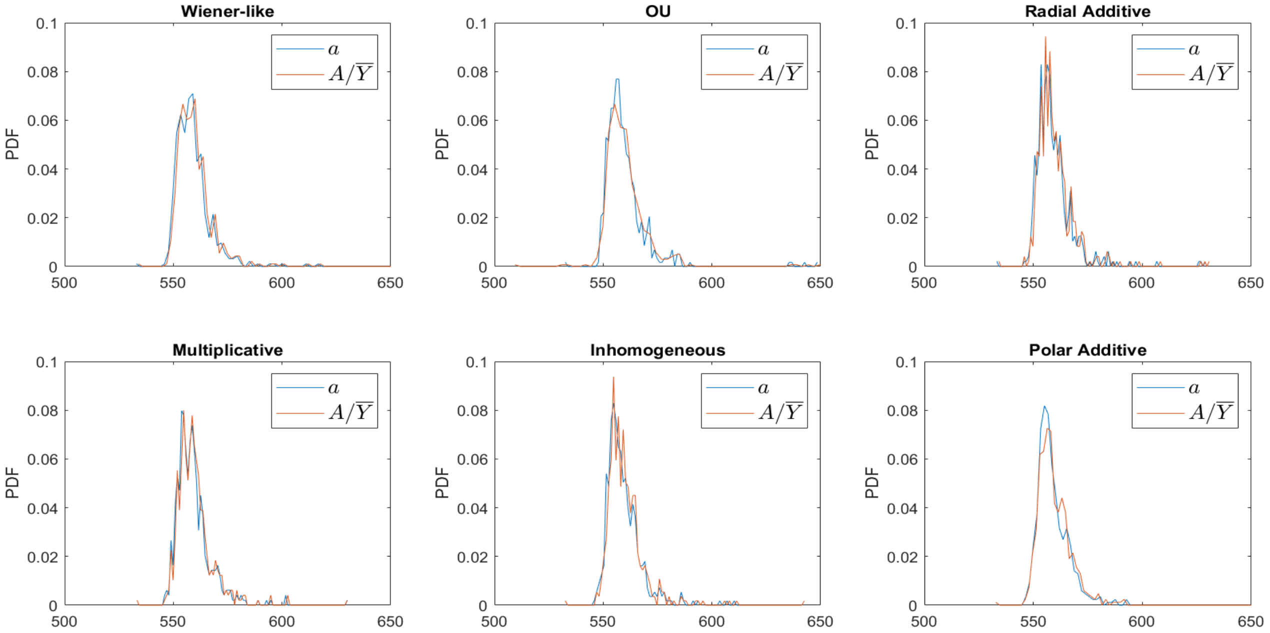

Figure 6 – Probability density functions of the head of the fire a and of the scaled burned area $A/\overline{Y}$ for all the six random fire front models at time $t \approx 700$.

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